All Methods SF

Simplex — Min $= [\text{Opti} \ (c_j - z_j) \geq 0]$ } Least +ve row.
— Max $= [\text{opti} \ (c_j - z_j \leq 0)]$

Dual Simplex [Conv.t mini to max by (-) ; $c_j - z_j$ & sol$^n \geq 0$.] If ! optical det^n table — only z. mini $-(c_j - z_j)$ — Row most — Col - least +ve

Big-M — ~~Min $[\text{opti} \ (c_j - z_j) \geq 0]$~~ +M } Least +ve ratio
— Max $[\text{opti} \ (c_j - z_j) \leq 0]$ −M

Two-phase — Min $A_1 + A_2$ — Phase I } $c_j - z_j = 0$ — Phase II } $c_j - z_j \geq 0$
— Max $(-A_1 - A_2)$ — Phase II $c_j - z_j \leq 0$

linear programming 8/01/2025

Graphical method :-

$\geq \leq$ ———

$> <$ - - - - -

$\leq$ - Towards origin

$\geq$ - outside

Q. $Z_{max} = 12x_1 + 16x_2$

subject to

$10x_1 + 20x_2 \leq 120$

$8x_1 + 8x_2 \leq 80$

x_1 & $x_2 \geq 0$

Step 1 :- form an equation

$10x_1 + 20x_2 = 120$ — 1

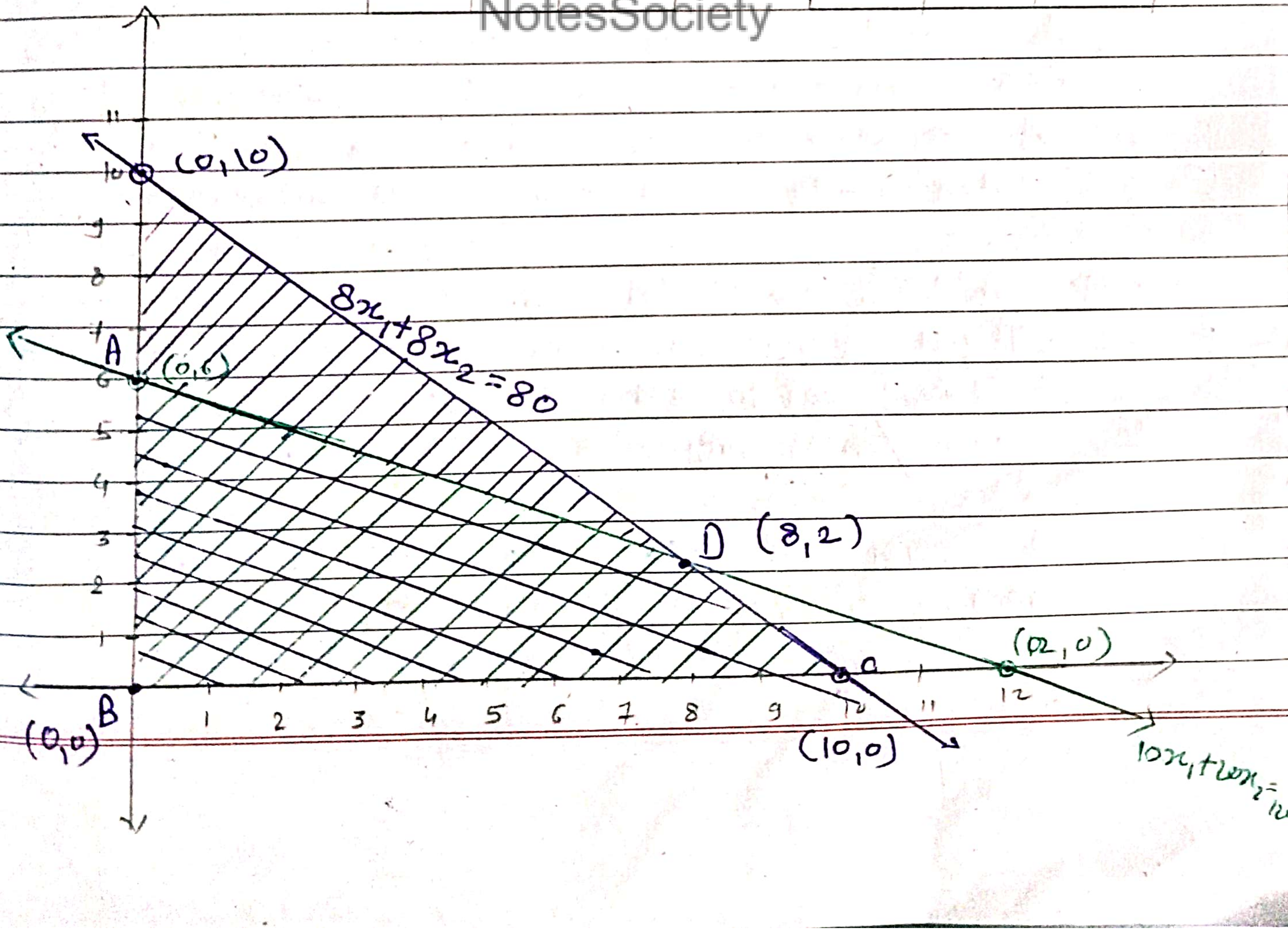
$8x_1 + 8x_2 = 80$ — 2

for ①

x_1	0	12
x_2	6	0

for ②

x_1	0	10
x_2	10	0



$A(0,6) = \quad Z(A) = 96$

$B(0,0) = \quad Z(B) = 0$

$C(10,0) = \quad Z(C) = 120$

$D(8,2) = \quad Z(D) = 128 \quad$ Max

$\therefore Z(D) = 128$ is the optimal solution

LPP Theory

- The Linear programming problem (LPP) is a problem that is concerned with finding the optimal value of the given linear function
- The optimal value can be either maximum value or minimum value. Here, the given linear function is considered an objective function
- The objective funⁿ can contain several variables which are subjected to the conditions and it has to satisfy the set of linear inequalities called linear constraints
- The linear programming problems can be used to get the optimal solution for the following scenarios, such as manufacturing problems, diet problems, transportation problems, allocation problems and so on

☆ Components of linear programming :

- A linear equation reflecting the aim to maximise or minimize, commonly stated as a combination of choice variables, is an objective function (e.g $Z = ax+by$)
- Variables representing the quantities to be determined or optimised in the issue are known as decision variabl..
- Constraints :- A collection of linear inequalities or equations that specify the decision variable's constraints or restrictions e.g ($cx+dy \geq e$, $px+qy \leq r$)
- The set of possible solⁿ is represented by the region defined by the intersection of all restrictions
- The best possible solⁿ. that maximises or minimises

the goal function
- Non-negativity the condition that decision variable be non-negative (greater than or equal to zero) is one of the constraints (eg $x \geq 0$ $y \geq 0$)

★ Characteristics of linear programming :
- Technique for mathematical optimization
- Linear equation to maximize or minimize the objective function.
- Decision variables: These are the quantities to be optimized
- Linear inequalities or equations serve as constraints
- Intersection of constraints defines the feasible region
- optimal option: The best algebraic approaches were employed
- It may be used in resource allocation, logistics, and other areas.
- It aids in decision-making and efficiency improvement
- mathematically models real-world problems

★ Advantages of LPP
- utilized for analyzing numerous military, social, economic social, and industrial problems.
- Linear programming is appropriate for solving complex problems.
- It assists in productive management of an organization for better outcomes.

Applications :

1) Resource Allocation :

L.P is extensively used in industries to allocate limited resources, such as labor, raw materials, & machine hours, to maximize production output while minimizing costs

2) Production Planning :

Manufacturing Companies use linear programming to plan their production schedules, taking into account constraints like available production time, labor

3) Transpotation and logistics.

4) Financial Portfolio optimization.

5) Marketing Campaigns

6) Agricultural Planning

7) Project scheduling

8) Diet Planning

9) Blending problems in manufacturing

10) Game Theory

Example

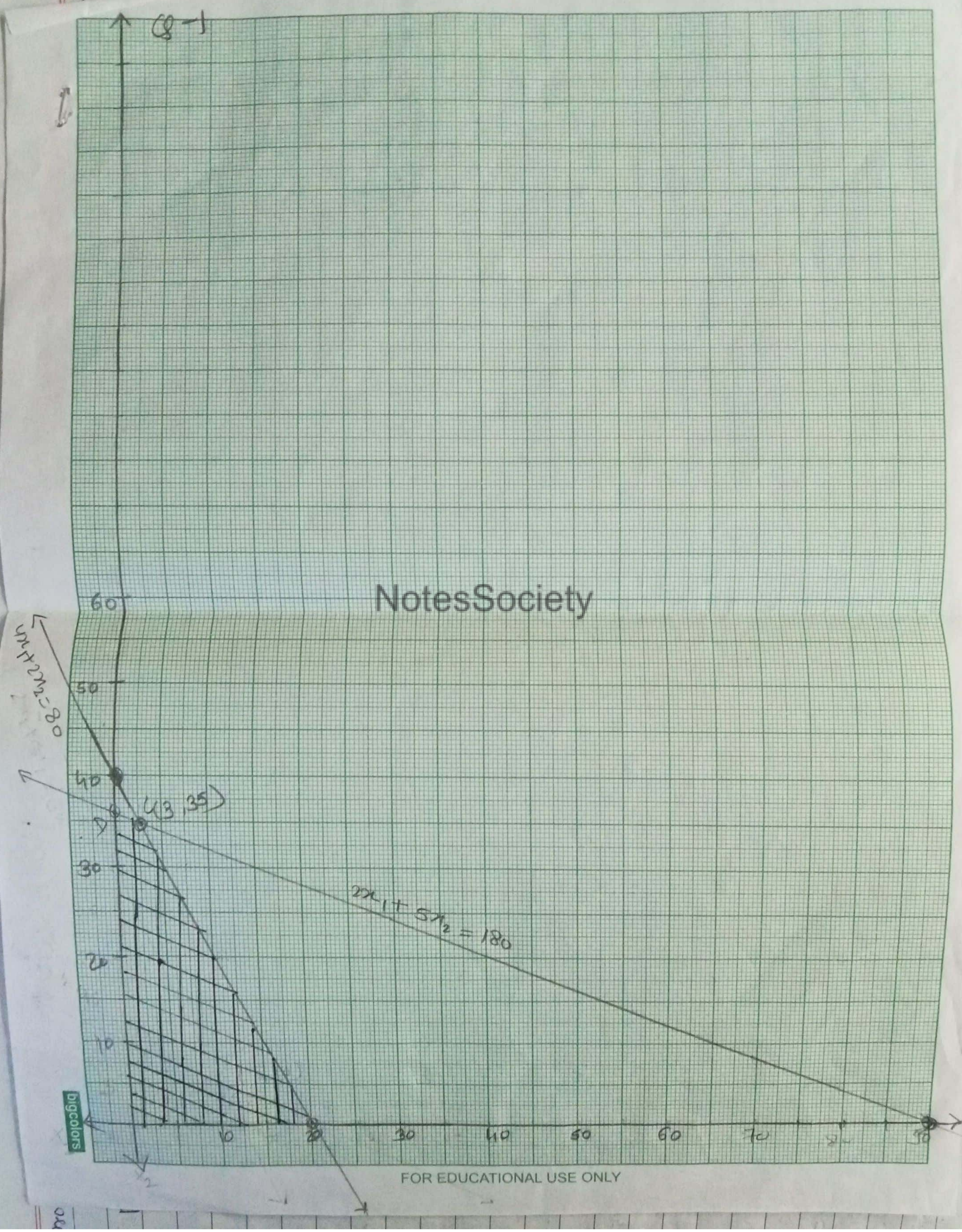
- Consider a chocolate manufacturing Company which produces only two types of chocolate - A and B Both the chocolate requires milk and Cocoa only. To manufacture each unit of A and B, following quantities are requiry

1. Each unit of A requires 1 unit of milk & 3 unit of Cocoa
2. Each unit of B requires 1 unit of milk & 2 units of Cocoa

The Company kitchen has a total of 5 units of milk & 12 units of Cocoa on each sale, the Company

Makes a profit of Rs 6 per unit A sold
Rs 5 per unit B sold

	Products A	Products B	Availability
Milk	1	1	~~5~~ 5
Cocoa	3	2	12
profit			


Q-1
60
50
40
30
20
10
C(3,35)
D
$4x_1+x_2=80$
$2x_1+5x_2=180$
10
20
30
40
50
60
70
90
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Q-1 A manufactures produces two types of models M_1 and M_2. Each, model of M_1 required 4 hr of grinding & 2 hr of polishing, whearre as each model of the type M2 required 2 hr of grinding and 5 hr of polishing

- The manufactures has 2 grinders and 3 polisher Each 3 polisher Each grinder work 40 hr a week and each polisher work for 60 hr a week
- A profit on M_1 model is ₹ 3:00 and an model M_2 is ₹ 4:00 whatever is produced in a week is solid in market
- How should the manufactures allocate his production capacity to the two types of models so that he may make the maximum profit in a week?

Model	M_1	M_2	Availability
grinding	4	2	40
polishing	2	5	60
profit	3	4	

$Z_{max} = 3x_1 + 4x_2$

Subject to $4x_1 + 2x_2 \le 80$ → for Two grinder

$2x_1 + 5x_2 \le 180$ for Three polisher

correct

- 2

$4x_1 + 2x_2 = 80$ — 1 $\quad$ $2x_1 + 5x_2 = 180$

x_1	0	20
x_2	40	0

x_1	0	90
x_2	36	0

$2x_2 = 80$

$x_2 = 80/2 = 40$

$4x_1 = 80$

$x_1 = 80/4 = 20$

$5x_2 = 180$

$x_2 = 180/5 = \cancel{12}\ 36$

$2x_2 = 180$

$x_2 = 180/2 = 90$

A (0, 0)

B (20, 0)

C (30, 35)

D (0, 36)

$Z_A = 3x_1 + 4x_2$
$= 3(0) + 4(0)$
$= 0$

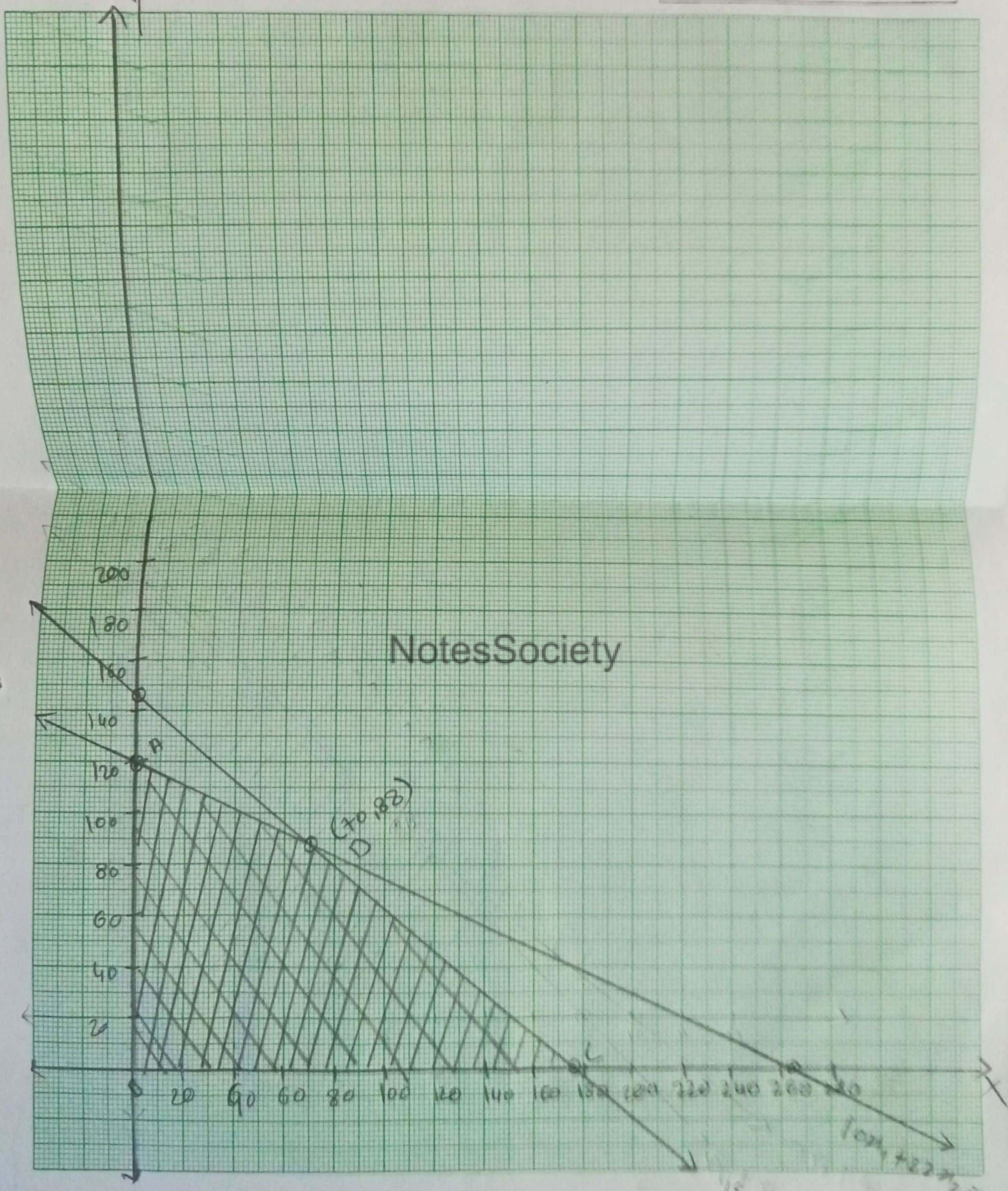
$Z_B = 3x_1 + 4x_2$
$= 3(20) + 4(0)$
$= 60$

$Z_C = 3x_1 + 4x_2$
$= 3(30) + 4(35)$
$= 90 + 140$
$= 230$

$Z_D = 3x_1 + 4x_2$
$= 3(0) + 4(36)$
$= 144$

$Z_C = 230$ is the optimal solution

on Y-axis 1 cm =



$10x_1+22x_2=264$..

$15x_1+18x_2=2640$

Origin = (   )

Intercepts

on X-axis = ... Slope = ...

on Y-axis = ...

Q-2 A Company make two products A and B. Both require processing on 2 machines product A take 10 & 15 min on the two machines per unit and product B take 22 & 18 min per unit on the two machines Both the machines are available for 2640 min per week The product are sold for ₹ 200 /- & ₹ 175 resp. per unit formulate a LP to maximize revenue ? The market can take a maximum of 150 units of product A

	Products		Availability
	A	B	
machine 1	10	22	2640
machine 2	15	18	2640
Profit	200	175	

$A \leq 150$

$Z_{max} = 200x_1 + 175x_2$

Subject to $10x_1 + 22x_2 \leq 2640$

$15x_1 + 18x_2 \leq 2640$

$A \leq 150$ $\quad x_1, x_1 \geq 0$

$10x_1 + 22x_2 = 2640$ — ①

$15x_1 + 18x_2 = 2640$ — ②

x_1	0	264
x_2	120	0

x_1	0	176
x_2	146.6	0

$22x_2 = 2640$

$x_2 = 2640/22$

$= 120$

$10x_1 = 2640$

$x_1 = 264$

$18x_2 = 2640$

$x_2 = 146.6$

$15x_1 = 2640$

$x_1 = 176$

$200x_1 + 175x_2$

A(0,120) $Z_{max} = 21000$

B(0,0) $Z_{max} = 0$

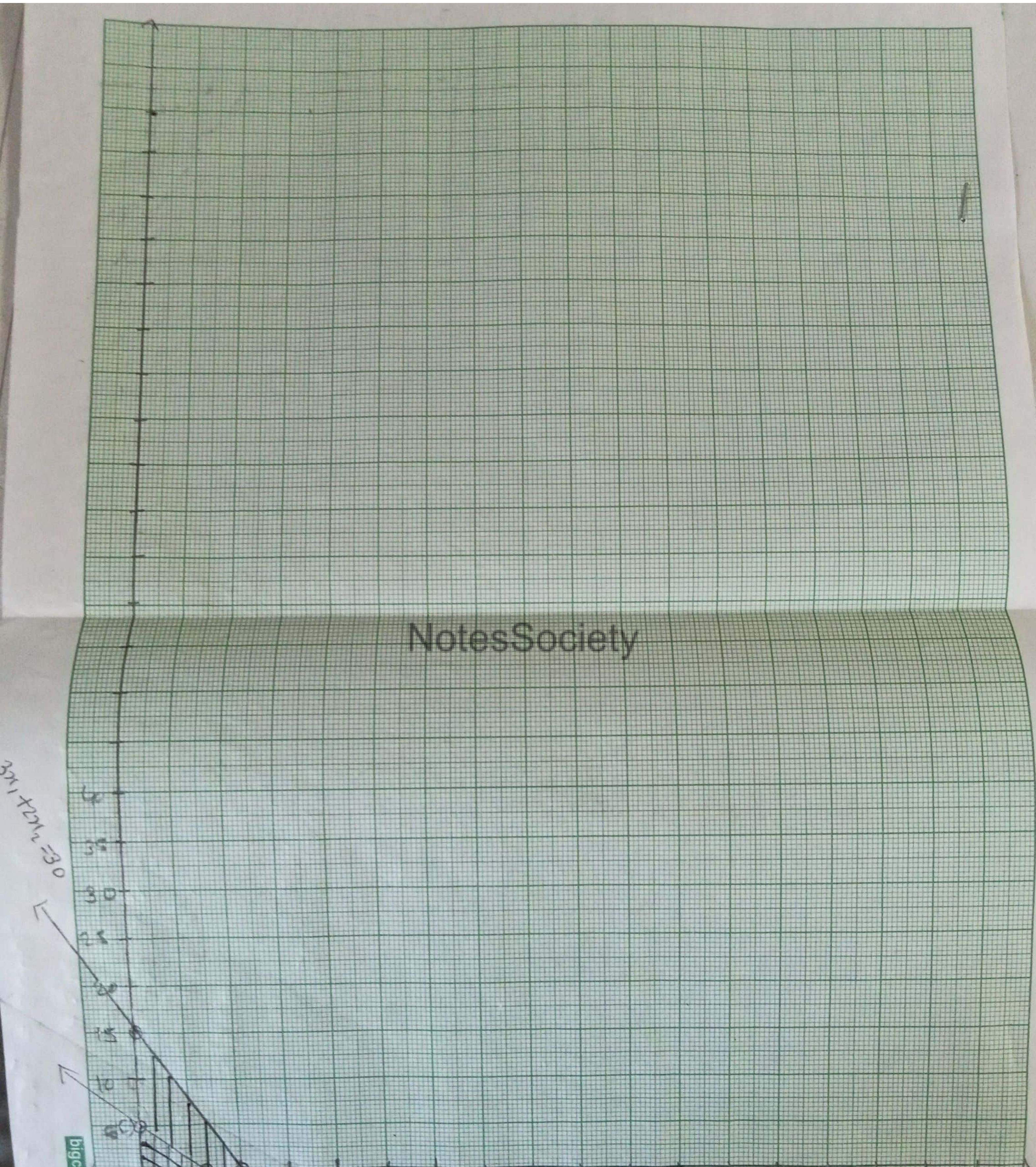
C(176,0) $Z_{max} = 35200$

D(70,88) $Z_{max} = 29400$

Z_{max} at C(176,0) = 35200 is the optimal solution.

Q-3 A shop can make two types of sweets A and B. They use two resources, flour & sugar. To make one packet of sweet A, they need 3 kg of flour & 4 kg of sugar. To make one packet of sweet B they need 2 kg of flour and 5 kg of sugar. They have 30 kg of flour & 25 kg of sugar. These sweets are sold at Rs 800 & Rs 900 per packet respectively. Find the best product mix so that total revenue will be maximum.

resources	sweets A	sweets B	Availability
flour	3	2	30
sugar	4	5	25
profit	800	900	


$3x_1+2x_2=30$
$4x_1+5x_2=25$
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$Z_{max} = 800x_1 + 900x_2$

subject to $3x_1 + 2x_2 \leq 30$ $\qquad$ $3x_1 + 2x_2 = 30$ — ①

$4x_1 + 5x_2 \leq 25$ $\qquad$ $4x_1 + 5x_2 = 25$ — ②

$x_1, x_2 \geq 0$

x_1	0	10
x_2	15	0

x_1	0	6.2
x_2	5	0

A (0,0)

B (6.2,0)

C (0,5)

$Z_{max} = 800x_1 + 900x_2$

$Z_A = 800(0) + 900(0)$

$= 0$

$Z_B = 800x_1 + 900x_2$

$= 800(6.2) + 900(0)$

$= 4960.0$

$Z_C = 800x_1 + 900x_2$

$= 800(0) + 900(5)$

$= 0 + 4500$

$= 4500.$

$\therefore Z_{max}$ at $Z_B = 4960$. is the optimal solution

Q-4 - A school is preparing a trip for 400 students. The Company who is providing transportation has 10 buses of 50 seats and 8 buses of 40 seats but only 9 drivers available. The rental cost for larger bus is 800 and 600 for small bus. Calculate How many buses of each Type should be used for the trip for the least possible Cost

[x_1 = small bus , x_2 = big bus]

	10 small bus	8 big bus	Availability
Seats	50	40	400
drivers	1	1	9
	800	600	

$Z_{min} = 800x_1 + 600x_2$

Total seats available ≥ 400

$50x_1 + 40x_2 \geq 400$

$x_1 + x_2 \leq 9$

$x_1, x_2 \geq 0$

$50x_1 + 40x_2 = 400$ — ①

x_1	0	8
x_2	10	0

$x_1 + x_2 = 9$ — ②

x_1	0	9
x_2	9	0

$40x_2 = 400$

$x_2 = 10$

$50x_1 = 400$

$x_2 = 9$

$x_1 = 9$

$x_1 = 40/5$
$x_1 = 8$

A(0,8)
B(5.3, 3.9)
C(0,10)

$Z_{min}(A) = 800x_1 + 600x_2$
$= 800(0) + 600(8)$
$= 4800$

$Z_{min}(B) = 800(5.3) + 600(3.9)$
$= 4240 + 2340$
$= 6580$

$Z_{min}(C) = 800(0) + 600(10)$
$= 6000$

Zmin at A $Z_A = (0,8)$ The optimal solution is

$Z_{min} = A(0,8) = 4800$

Q-5 A store wants to liquidate 200 shirts and 100 pairs of pants from last session They have decided to put together two offers A and B offer A is a package of one shirt and a pair of pants which will sell for 30 offer B is a package of three shirts and a pair of pants which will sell for 50 The store does not want to sell less than 20 packages of offer A and less than 10 of offer B. How many packages of each do they have to deal to maximize the money generated from promotion

Hint - x_1 - no. of packages for A, x_2 - no. of packages for

	offers		Availability
Product	A	B	
Shirt	1	3	$\leq$ 200
Pant	2	2	$\leq$ 200
Sell	30	50	

$x_1 + 3x_2 \leq 200$

$2x_1 + 2x_2 \leq 200$

$A_1 \leq 20$

$B_1 \leq 10$

$Z_{max} = 30x_1 + 50x_2$

$Z_{max} = 30x_1 + 50x_2$

subject to

$x_1 + 3x_2 \leq 10$

$B_1 \leq 10$ $\quad 2x_1 + 2x_2 \leq 20$

$A_1 \leq 20$ $\quad x_1, x_2 \geq 0$

Here. $x_1 + 3x_2 = 200$ — (1) $\quad 2x_1 + 2x_2 = 200$ — 2

x_1	0	200
x_2	66.6	0

x_1	0	100
x_2	100	0

$3x_2 = 200$

$x_2 = 200/3$

$= 66.6$

$x_1 = 200$

$2x_2 = 200$

$x_2 = 200/2 = 100$

$x_1 = 100$

A (0, 10)   C (20, 0)

B (0, 0)   D (20, 10)

$Z_{max} = 30x_1 + 50x_2$

$Z_A = 30(0) + 50(10)$
$= 500$

$Z_C = 30(20) + 50(0)$
$= 600$

$Z_B = 30(0) + 50(0)$
$= 000.$

$Z_D = 30(20) + 50(10)$
$= 600 + 500$
$= 1100.$

$Z_{max} = 1100 = Z_D$

The optimal solution ~~of~~ is Zmin B·(20, 10) = 1100.

Key Column = Max +ve number.
Key row = Min +ve number

★ Simplex method :- It is used for an maximization problem

$$Z_{max} = C_j - Z_j \leq 0$$

Q - $Z = 12x_1 + 16x_2$

$10x_1 + 20x_2 \leq 120$

$8x_1 + 8x_2 \leq 80$

$x_1, x_2 \geq 0$

eqn in standard form is,

$Z = 12x_1 + 16x_2 + 0S_1 + 0S_2$

$10x_1 + 20x_2 + S_1 + 0S_2 = 120$

$8x_1 + 8x_2 + 0S_1 + S_2 = 80$

$Z_j = \Sigma C_{Bj} \times A_{kj}$

$C_j - Z_j \leq 0$

Table for optimization solution

CBj	Cj	12	16	0	0		
	BV	X_1	X_2	S_1	S_2	Soln	Ratio
0	S_1	10	$\boxed{20}$	1	0	120	$\frac{120}{20} = 6$ ← Min
0	S_2	8	8	0	1	80	$\frac{80}{8} = 10$
	Zj	0	0	0	0		
	~~Zj - Cj~~ Cj - Zj	12	16 ↑ Max	0	0.		

I/p var : val = $\frac{\text{old value}}{\text{key value}}$

Remaining var = old value - $\frac{(\text{key Col}^n \times \text{key row})}{\text{key value}}$

Iteration 1st

	C_j	12	16	0	0		
C_{Bj}	X_b	x_1	x_2	S_1	S_2	Sol^n	Ratio
16	x_2	$\frac{1}{2}$	1	$1/20$	0	6	$\frac{6}{\frac{1}{2}} = \frac{6}{1} \times 2 = 12$
0	S_2	$\boxed{4}$	0	$-2/5$	1	32	$\frac{32}{4} = 8$ ←
	z_j	8	16	$4/5$	0		
	$C_j - z_j$	4 ↑	0	$-4/5$	0		

Iteration 2nd

	C_j	12	16	0	0		
C_B	X_b	X_1	X_2	S_1	S_2	Solution	Ratio
16	x_2	0	1	$1/10$	$-\frac{1}{8}$	2	
12	X_1	1	0	$-1/10$	$1/4$	8	
	Z_j	12	16	$\frac{2}{5}$	1	128	
	$C_j - Z_j$	0	0	$-\frac{2}{5}$	-1		

$$Z_{max} = 12x_1 + 16x_2$$
$$= 12(8) + 16(2)$$
$$= 96 + 32$$
$$= 128$$

H.W.

Q $Z = x_1 + 2x_2$

subject to,

$$x_1 - 3x_2 \leq 1$$
$$-x_1 + 2x_2 \leq 4$$
$$x_1 \,\&\, x_2 \geq 0.$$

→ Adding slack variable into equation

$$\text{Max } z = x_1 + 2x_2 + 0S_1 + 0S_2$$
$$x_1 - 3x_2 + S_1 + 0S_2 = 1$$
$$-x_1 + 2x_2 + 0S_1 + S_2 = 4$$

where S_1, S_2 slack variables

Initial simplex method

C_{Bj}	C_j	1	2	0	0	Solution	Ratio
	B.V	X_1	X_2	S_1	S_2		
0	S_1	$\boxed{1}$	-3	1	0	1	1 ←
0	S_2	$\boxed{-1}$	2	0	1	4	$\frac{4}{-1} = -4$
	Z_j	0	0	0	0		
	$C_j - Z_j$	1	2 ↑	0	0.		

$$\text{New value} = \text{old value} - \frac{\text{Corresponding row value} \times \text{Corresponding Column value}}{\text{Key value}}$$

II Iteration

C_{Bj}	C_j	1	2	0	0		
	B.V.	X_1	X_2	S_1	S_2	Solution	Ratio
1	X_1	$-1/3$	1	$-1/3$	0	$-1/3$	$-\frac{1}{3} = -\frac{1}{3}$
0	S_2	$1/3$	0	$2/3$	1	$14/3$	$\frac{5}{1} = 5$ ←
	Z_j	$-1/3$	$+1$	$-1/3$	0		
	$C_j - Z_j$	$4/3$	1	$+1/3$	0		

III Iteration :-

C_{Bj}	C_j	1	2	0	0		
	B.V	X_1	X_2	S_1	S_2	Solution	Ratio
1	X_1	1	0	4	0	16	
2	X_2	0	1	1	0	5	
	Z_j	1	2	6	0	26	
	$C_j - Z_j$	0	0	-6	0		

$$Z_{max} = x_1 + 2x_2$$
$$= 16 + 2(5)$$
$$= 16 + 10$$
$$= 26$$

is the optimal solution

Home Work

② $Z = x_1 + 3x_2$

subject to,

$-x_1 + x_2 \leq 20$

$-2x_1 + x_2 \leq 50$

$x_1, x_2 \geq 0.$

-> $Z = x_1 + 3x_2 + 0S_1 + 0S_2$

$-x_1 + x_2 + S_1 + 0S_2 = 20$

$-2x_1 + x_2 + 0S_1 + S_2 = 50$

where S_1, S_2 are slack Variables.

CBj	Cj	1	3.	0	0		
	B.V	X_1	X_2	S_1	S_2	Solution	Ratio
0	S_1	-1	$\boxed{1}$	1.	0	20	$\frac{20}{+1} = 20$ ←
0	S_2	-2	1	0	1	50	$\frac{50}{1} = 50$
	Z_j	0	0	0	0		
	$C_j - Z_j$	1	3 ↑	0	0		

Iteration II

	Cj	1	3	0	0		
CBj	B.V	X_1	X_2	S_1	S_2	Solution	Ratio
3	X_2	-1	1	1	0.	20.	$\frac{20}{-1} = -20$
0	S_2	-1	0	-1	1	30	$\frac{30}{-1}$ -30.
	Z_j	-3	3	3	0.		
	$C_j - Z_j$	4 ↑	0.	-3	0		

Ratio is not Positive so Not Fisible solution.

Minimization problem solving with Simplex method

optimality mini = $C_j - Z_j \geq 0$

step(I) – for getting optimum sol^n we need to convert all $\geq$ sign inequation in $\leq$

– for that multiply by -1 to inequality fun^n which having $\geq$ sign.

e.g. $2x_1 + 3x_2 \geq 5$ — (I)

multiply by -1 to eq^n (I) on both sides

$\therefore -2x_1 - 3x_2 \leq -5$

step II add slack variables in eq^n

step III :- Calculate basic variable of $C_j - Z_j$ & check optimality

step IV. for iteration table 1

key Col^m = Most $-ve$ value.

key Row = Ratio → least $+ve$ value.

key value = Intersection point value of Row & Col^m

New value for S_1 / S_2 = old value − $\left(\frac{\text{key } Col^n \text{ value} * \text{keyrow value}}{}\right)$

All the rest of steps are same till

$C_j - Z_j \geq 0$

Maximization

$\text{Max } Z = 3x_1 + 2x_2$

$x_1 + x_2 \le 4$

$x_1 - x_2 \le 2$

$x_1 \;\&\; x_2 \ge 0$

$\rightarrow Z = 3x_1 + 2x_2 + 0S_1 + 0S_2$

$x_1 + x_2 + S_1 + 0S_2 = 4$

$x_1 - x_2 + 0S_1 + S_2 = 2$

CB_i	C_j	3	2	0	0		
	BV	X_1	X_2	S_1	S_2	solution	Ratio.
0	S_1	1	1	1	0	4	$\frac{4}{1} = 4$
0	S_2	1	-1	0	1	2	$\frac{2}{1} = 2$ ←
	Z_j	0	0	0	0		
	$C_j - Z_j$	3 ↑	2	0	0		

Iteration II

	C_j	3	2	0	0		
CB_j	B.V	X_1	X_2	S_1	S_2	Sol^n	Ratio.
0	S_1	0	2	1	-1	2	$\frac{2}{2} = 1$ ←
3	X_1	1	-1	0	1	2	$\frac{2}{-1} = -2$
	Z_j	3	-3	0	3		
	$C_j - Z_j$	0	5 ↑	0	-3		

Iteration II

	C_j	3	2	0	0		
C_{Bj}	B.V	x_1	x_2	s_1	s_2	sol^n	Ratio
2	x_2	0	1	1/2	-1/2	1	
3	x_1	1	0	1/2	1/2	3	
	Z_j	3	2	5/2	1/2	11	
	$C_j - Z_j$	0	0	$-\frac{5}{2}$	$-\frac{1}{2}$		

$\therefore \quad C_j - Z_j \leq 0$

$$\therefore \quad \text{Max } Z = 3x_1 + 2x_2$$
$$= 3(3) + 2(1)$$
$$= 9 + 2$$
$$= 11$$

$\therefore$ The optimal solution is $x_1 = 3$, & $x_2 = 1$

good

Minimization

Q $Z = 2x_1 - 3x_2 + 6x_3$

Subject to $3x_1 - x_2 + 2x_3 \leq 7$

$2x_1 + 4x_2 \geq 12$

$-4x_1 + 3x_2 + 8x_3 \leq 10$

$x_1, x_2, x_3 \geq 0$

$\rightarrow$ $Z = 2x_1 - 3x_2 + 6x_3 + 0s_1 + 0s_2 + 0s_3$

$3x_1 - x_2 + 2x_3 + s_1 = 7$

Multiply by (-1) $-2x_1 - 4x_2 + s_2 = 12$

$-4x_1 + 3x_2 + 8x_3 + s_3 = 10$

CBj	Cj	2	-3	6	0	0	0		
	B.V.	x_1	x_2	x_3	s_1	s_2	s_3	sol^n	Ratio
0	s_1	3	-1	2	1	0	0	7	$\frac{7}{-1} = -7$
0	s_2	-2	-4	0	0	1	0	12	$\frac{12}{-4} = -3$
0	s_3	-4	3	8	0	0	1	10	$\frac{10}{3} = 3.3$
	Z_j	0	0	0	0	0	0		
	$C_j - Z_j$	+2	-3	6	0	0	0		

C_{Bj}	C_j	2	-3	6	0	0	0		
	B.V	X_1	X_2	X_3	S_1	S_2	S_3	solution	Ratio
0	S_1	$\boxed{5/3}$	0	14/3	1	0	1/3	31/3	$\frac{31}{5}$
0	S_2	$-\frac{22}{3}$	0	$\frac{32}{3}$	0	1	4/3	76/3	$-\frac{76}{22} = -\frac{38}{11}$
-3	X_2	-4/3	1	8/3	0	0	1/3	10/3	$-\frac{10}{4}$
	Z_j	4	-3	-8	0	0	-1		
	$C_j - Z_j$	-2	0	14	0	0	1		

Iteration II

	C_j	2	-3	6	0	0	0		
C_{Bj}	B.V.	X_1	X_2	X_3	S_1	S_2	S_3	solution	ratio.
2	X_1	1	0	14/5	3/5	0	1/5	31/5	
0	S_2	0	0	$\frac{256}{5}$	22/5	1	14/5	354/5	
-3	X_2	0	1	32/5	4/5	0	9/5	58/5	
	Z_j	2	-3	$-\frac{68}{5}$	-6/5	0	-7/5	$\frac{-112}{5}$	
	$C_j - Z_j$	0	0	$\frac{98}{5}$	6/5	0	7/5		

H.w maximization Home work

$z = x_1 + 2x_2$

→ Initial table

CBj	Cj	1	2	0	0		Ratio
	B.V	X_1	X_2	S_1	S_2	Sol^n	
0	S_1	1	-3	1	0	1	$1/-3 = -$
0	S_2	-1	2	0	1	4	$4/2 = 2$ ←
	Z_j	0	0	0	0		
	$C_j - Z_j$	1	2 ↑	0	0		

	Cj	1	2	0	0		
CBj	B.V	X_1	X_2	S_1	S_2	Sol^n	Ratio
0	S_1	$-1/2$	0	1	$-3/2$	~~7~~	$\frac{-5}{-1/2} = \frac{-5}{2}$ ←
2	X_2	$-1/2$	1	~~1~~0	$1/2$	2	$\frac{2}{-1/2} = -1$
	Z_j	-1	2	2	1		
	$C_j - Z_j$	2 ↑	0	-2	-1		

CBj	Cj	1	2	0	0		
	B.V	x_1	x_2	s_1	s_2	Solution	Ratio
1	x_1	1	0	-2	3	10	
2	x_2	0	1	0	2	7	
	Zj	1	2	0	7	24	
	Cj-Zj	0	0	0	-7		

$\therefore \quad C_j - Z_j \leq 0$.

Here $x_1 = 10 \quad x_2 = 7$

$$
\begin{aligned}
Z_{max} &= x_1 + 2x_2 \\
&= 10 + 2(7) \\
&= 10 + 14 \\
&= 24
\end{aligned}
$$

optimal solution $x_1 = 10, \quad x_2 = 7 \qquad = 24$

+/ No − (Cj − zj)

Minimizⁿ = least +ve maxzⁿ = most −ve

★ Dual Simplex method :- $c_j - z_j \geq 0$ and solⁿ

Step (I) Convert minimization problᵐ into maxzⁿ $\geq$ to $\leq$ b_j -

Step (II) add slack variable

Step (III) Complete Basic table with $c_j - z_j$ if $c_j - z_j$ & solⁿ will not optimal Calculate Determination of entering variable

Step (IV) :: Find most −ve value for leaving variable

Step (V) :- Determination of Entering variable

Variables x_1 x_2 s_1 s_2 ...

$-(c_j - z_j$

Q. Min $Z = x_1 + 2x_2 + 3x_3$

Sub to $2x_1 - x_2 + x_3 \geq 4$ −(I)

$x_1 + x_2 + 2x_3 \leq 8$ −(II)

$x_2 - x_3 \geq 2$ −(III)

multiply eqⁿ ① & ③ by −1

$-2x_1 + x_2 - x_3 \leq -4$

$-x_2 + x_3 \leq -2$

Add slack variable to the equality

$-2x_1 + x_2 - x_3 + s_1 = -4$

$x_1 + x_2 + 2x_3 + s_2 = 8$

$-x_2 + x_3 + s_3 = -2$

C_{Bj}	c_j	1	2	3	0	0	0	
	B.V.	x_1	x_2	x_3	s_1	s_2	s_3	solution
0	s_1	−2	+1	−1	1	0	0	−4
0	s_2	1	1	2	0	1	0	8
0	s_3	0	−1	1	0	0	1	−2
	z_j	0	0	0	0	0	0	
	$c_j - z_j$	1	2	3	0	0	0	
	$-(c_j - z_j)$	−1	−2	−3	0	0	0	
	s_1	−2	−1	−1	1	0	0	
	Ratio	1/2	2	3	0	0	0	

C_{Bj}	c_j	1	2	3	0	0	0	
	B.V.	x_1	x_2	x_3	s_1	s_2	s_3	solution
1	x_1	1	−1/2	1/2	−1/2	0	0	2
0	s_2	0	3/2	3/2	+1/2	1	0	7
0	s_3	0	−1	1	0	0	1	−2
	z_j	1	−1/2	1/2	−1/2	0	0	2
	$c_j - z_j$	0	5/2	5/2	1/2	0	0	
	$-(c_j - z_j)$	0	−5/2	−5/2	−1/2	0	0	
	~~ratio~~ s_3	0	−1	+1	0	0	1	−2
	ratio	0	5/2	−5/2	0	0	0	

$\cancel{-2x_1 + x_2 - x_3 + 3}$

C_{Bj}	C_j	1	2	3	0	0	0	solution
	B.V	X_1	X_2	X_3	S_1	S_2	S_3	
1	X_1	1	0	0	$-1/2$	0	$-1/2$	3
0	S_2	0	0	3	$1/2$	1	$3/2$	3
2	X_2	0	1	-1	0	0	-1	2
	Z_j	1	2	-2	$-1/2$	0	$-5/2$	7
	$C_j - Z_j$	0	0	5	$1/2$	0	$5/2$	

Determination table

$\therefore C_j - Z_j \geq 0$

$\therefore$ Solution $\cancel{>0}$

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$$Z_{min} = x_1 + 2x_2 + 3x_3$$
$$= 3 + 2(2) + 3(0)$$
$$= 3 + 4 + 0$$
$$= 7$$

Maximization with dual

$*$ No $-(c_j - z_j)$ Sol^n

optimality $c_j - z_j \leq 0$

① . Max $z = -3x_1 - 2x_2$

subject to
$$x_1 + x_2 \geq 1 \quad -①$$
$$x_1 + x_2 \leq 7 \quad -②$$
$$x_1 + 2x_2 \geq 10 \quad -③$$
$$+x_2 \leq 3 \quad -④$$

∴ Here multiply by -1 to eq^n ① & ③

$$-x_1 - x_2 + s_1 = 1 \quad -①$$
$$x_1 + x_2 + s_2 = 7 \quad -②$$
$$-x_1 - 2x_2 + s_3 = -10 \quad -③$$
$$-x_2 = 3 \quad -④$$

CB_j	C_j	-3	-2	0	0	0	0	
	B.V.	x_1	x_2	$\cancel{x_3}$ S_1	S_2	S_3	S_4	Sol^n
0	S_1	-1	-1	1	0	0	0	-1
0	S_2	1	1	0	1	0	0	7
0	S_3	-1	-2	0	0	1	0	-10
0	S_4	0	$+1$	0	0	0	1	3
	z_j	0	0	0	0	0	0	
	$c_j - z_j$	-3	-2	0	0	0	0	

Determination table

$c_j - z_j$	-3	-2	0	0	0	0
S_3	-1	-2	0	0	1	0
Ratio	3	1	0	0	0	0

↑

C_{Bj}	C_j	-3	-2	0	0	0	0	Sol^n
	B_v	x_1	x_2	s_1	s_2	s_3	s_4	
0	s_1	$-1/2$	0	1	0	$-1/2$	0	4
0	s_2	$1/2$	0	0	1	$1/2$	0	2
-2	x_2	$1/2$	1	0	0	$-1/2$	0	5
0	s_4	$-1/2$	0	0	0	$+1/2$	1	-2
	z_j	-1	-2	0	0	1	0	-10
	$C_j - z_j$	-2	0	0	0	0	0	

Determination table.

$C_j - z_j$	-2	0	0	0	0	0
	$-1/2$	0	0	0	$1/2$	1
Ratio	4	-	-	-	-	-

C_{Bj}	c_j	-3	-2	0	0	0	0	
	B_v	x_1	x_2	s_1	s_2	s_3	s_4	Sol^n
0	s_1	0	0	1	0	-1	-1	6
0	s_2	0	0	0	1	1	1	0
-2	x_2	0	1	0	0	0	1	3
-3	x_1	1	0	0	0	-1	-2	4
	z_j	-3	-2	0	0	3	6	-18
	$C_j - z_j$	0	0	0	0	0	0	

$$Max\ z = -3x_1 - 2x_2$$
$$= -3(4) - 2(3)$$
$$= -12 - 6$$
$$= -18$$

☆ Big M - Method.

- It is extended Method of simplex algorithm
- It is also called penalty cost method.
- It contain two types.

1) maximization 2) Minimization

- "≤" add slack "≥" ~~add~~ subtract surplus & Add Artificial variable
- If sign is '=' then only add Artificial variab

e.g $x_1 + x_2 = 4$

$= x_1 + x_2 + A_2 = 4$

Artificial variable :-

- In Big M cost of artificial variable is always M in
- suppose ob. funⁿ is of minimization

e.g Minimization $z = Bx_1 + Bx_2$

let suppose 2 artificial var. A_1 & A_2 be needed to form a basis then change the objective funⁿ

minimize $z = Bx_1 + Bx_2 + MA_1 + MA_2$

suppose obj. funⁿ is of maximization

e.g Maximize $z = Bx_1 + Bx_2 + MA_1 + MA_2$

- Suppose obj funⁿ. of Maxⁿ

e.g Max $z = Bx_1 + Bx_2 - MA_1 - MA_2$

for maximizⁿ select the

Max. vax = $c_j - z_j$

minimum ratio = +ve

optimality

$c_j - z_j \le 0$

for mini

min value of $c_j - z_j$

~~Max~~ minimum Ratio

· $c_j - z_j \ge 0$.

Q. Maxi $z = 3x_1 - x_2$

Sub to $2x_1 + x_2 \geq 2$

$x_1 + 3x_2 \leq 3$

$x_2 \leq 4$

$x_1, x_2 \geq 0$

$c_j - z_j \leq 0$

write row only entering variable

→ $z = 3x_1 - x_2 - 0S_1 + 0S_2 + 0S_3 - MA_1$

$2x_1 + x_2 - S_1 + A_1 = 2$

$x_1 + 3x_2 + S_2 = 3$

$x_2 + S_3 = 4$

C_{Bj}	C_j	3	-1	0	0	0	-M		
	B.V	X_1	X_2	S_1	S_2	S_3	A	Solution	Ratio
-M	A_1	$\boxed{2}$	1	-1	0	0	1	2	$\frac{2}{2} = 1$
0	S_2	1	3	0	1	0	0	3	$\frac{3}{1} = 3$
0	S_3	0	1	0	0	1	0	4	$\frac{4}{0} = 0$
	Z_j	-2M	-M	M	0	0	-M		
	$C_j - Z_j$	3+2M	-1+M	-M	0	0	0		
		↑							

Iteration 1

	c_j	3	-1	0	0	0	-M	Solution	Ratio
C_{Bj}	B.V	x_1	x_2	S_1	S_2	S_3	A		
3	x_1	1	1/2	-1/2	0	0	–	1	-2
0	S_2	0	5/2	1/2	1	0	–	2	4
0	S_3	0	1	0	0	1	–	4	0.
	z_j	3	3/2	-3/2	0	0			
	$c_j - z_j$	0	-5/2	3/2	0	0			

	c_j	3	-1	0	0	0	Solution	Ratio
C_B	B.V.	x_1	x_2	S_1	S_2	S_3		
3	x_1	1	3	0	1	0	3	
0	S_1	0	5	1	2	0	4	
0	S_3	0	1	0	0	1	4	
	z_j	3	9	0	3	0	9	
	$c_j - z_j$	3	9	0	3	0		

$\therefore\ x_1 = 3 \quad x_2 = 0$

$Z = 3x_1 - x_2$

$= 3(3) - 0$

$= 9$

① $\text{Max } z = x_1 - x_2 + 3x_3$

Sub to $x_1 + x_2 \leq 20$

$x_1 + x_3 = 5$

$x_2 + x_3 \geq 10$

$x_1, x_2, x_3 \geq 0$

③ $\text{Min } z = 3x_1 + 2x_2 + 6x_3$

$4x_1 + x_2 + x_3 \leq 100$

$x_1 + x_2 \geq 40$

$x_1 + x_2 \leq 30$

② $\text{Min } z = x_1 - 3x_2 + 2x_3$

$3x_1 - x_2 + 2x_3 \leq 7$

$-2x_1 + 4x_2 \leq 12$

$-4x_1 + 3x_2 + 8x_3 \geq 10.$

④ $\text{Min } z = 3x_1 + 2x_2 + 7x_3$

$5x_1 + 2x_2 + 7x_3 = 420$

$3x_1 + 2x_2 + 5x_3 \geq 204$

$x_1, x_2, x_3 \geq 0$

①

$\rightarrow$ $\text{Max } z = x_1 - x_2 + 3x_3 - MA_1 - MA_2 + 0S_1 + 0S_2$

$x_1 + x_2 + S_1 = 20$

$x_1 + x_3 + A_1 = 5$ $\quad x_2 + x_3 + S_3 + A_3 = 10$

$x_2 + x_3 - S_2 + A_2 = 10$

CBj	Cj	1	-1	3	0	0	-M	-M		
	BV.	x_1	x_2	x_3	S_1	S_2	A_1	A_2	Solution	Ratio.
0	S_1	1	1	0	1	0	0	0	20	$\frac{20}{0} = \infty$
-M	A_1	1	0	$\boxed{1}$	0	0	1	0	5	$\frac{5}{1} = \boxed{5}$
-M	A_2	0	1	1	0	-1	0	1	10.	$\frac{10}{1} = 10.$
	Zj	-M	-M	-2M	0	M	-M	-M		
	Cj-Zj	1+M	-1+M	3+2M	0	-M	0	0		

↑

CBj	cj	1	-1	3	0	0	-M	-M		
	B.V	x_1	x_2	x_3	s_1	s_2	A_1	A_2	sol^n	Ratio
0	s_1	1	1	0	1	0		0	20	$\frac{20}{1} = 20$
3	x_3	1	0	1	0	0		0	5	$\frac{5}{0} = -$
-M	A_2	-1	$\boxed{1}$	0	0	-1		1	5	$\frac{5}{1} = 5$
	z_j	3+M	-M	3	0	M	3+M	-M		
	$c_j - z_j$	-2-M	-1+M	0	0	-M	-2M-3	0		

CBj	cj	1	-1	3	0	0	-M	-M		
	B.V	x_1	x_2	x_3	s_1	s_2	A_1	A_2	sol^n	Ratio
0	s_1	2	0	0	1	1	-	-	15	
3	x_3	1	0	1	0	0	-	-	5	
-1	x_2	-1	1	0	0	-1	-	-	5	
	z_j	4	-1	3	0	1			10	
	$c_j - z_j$	-3	0	0	0	-1	-	-		

$\therefore c_j - z_j \leq 0$

$z = x_1 - x_2 + 3x_3$

$= 0 - 5 + 3(5)$

$= -5 + 15$

$= 10$

(2) Min $Z = x_1 - 3x_2 + 2x_3 + 0S_1 + 0S_2 + 0S_3 + MA_1$

$3x_1 - x_2 + 2x_3 + S_1 = 7$

$-2x_1 + 4x_2 + S_2 = 12$

$-4x_1 + 3x_2 + 8x_3 - S_3 + A_1 = 10$

CBj	Cj	1	-3	2	0	0	0	M		
	B.V	X_1	X_2	X_3	S_1	S_2	S_3	A_1	Solution	Ratio
0	S_1	3	-1	2	1	0	0	0	7	$\frac{7}{-1} = -7$
0	S_2	-2	$\boxed{4}$	0	0	1	0	0	12	$\frac{12}{4} = 3$
M	A_1	-4	3	8	0	0	-1	1	10	$\frac{10}{3} = 3.3$
	Z_j	-4M	3M	8M	0	0	-M	M		
	$C_j - Z_j$	1+4M	-3-3M	2-8M	0	0	M	0		

CBj	Cj	1	-3	2	0	0	0	M		
	B.V	X_1	X_2	X_3	S_1	S_2	S_3	A_1	Solution	Ratio
0	S_1	5/2	0	2	1	1/4	0	0	10	$\frac{10}{5/2} = 4$ ←
-3	X_2	-1/2	1	0	0	1/4	0	0	3	$\frac{3}{-1/2} = -$
M	A_1	5/2	0	8	0	-3/4	-1	1	1	$\frac{1}{5/2} = \frac{2}{5}$ ←
	Z_j	$\frac{3+5M}{2}$	-3	8M	0	$\frac{-3(1+M)}{4}$	-M	M		
	$C_j - Z_j$	$\frac{-1+5M}{2}$	0	2-8M	0	$\frac{3(1+M)}{4}$	M	0		

CBj	Cj	1	-3	2	0	0	M	M		
	B.V	x_1	x_2	x_3	S_1	S_2	S_3	A_1	Solution	Ratio
0	S_1	0	0	6	1	1	1	–	9	$\frac{9}{1} = 9$
-3	x_2	0	1	8/5	0	17/80	-1/5	–	1/5	$\frac{1}{5} \mid -\frac{1}{5} = -1$
M	x_1	1	0	16/5	0	$-\frac{3}{10}$	$-\frac{2}{5}$	–	2/5	$\frac{2}{5} \mid -\frac{2}{5} = -1$
	z_j	1	-3	-8/5	0	$-\frac{301}{800}$	1/5			
	$c_j - z_j$	0	0	18/5	0	$+\frac{301}{800}$	-1/5			

CBj	Cj	1	-3	2	0	0	0	M		
	B.V	x_1	x_2	x_3	S_1	S_2	S_3	A_1	Solution	Ratio.
0	S_3	5/2	0	14	1	1/4	0	–	10	
-3	x_2	-1/2	1	0	0	19/80	0	–	0	
1	x_1	-5/2	0	-8	0	3/4	1	–	-1	
	z_j	-8/2	-3	-8	0					

* Two phase - Method :- for minin

- Consider basic var as 0 e.g $x_1, x_2 = 0$ - phase I
- $A_1, A_2 \rightarrow x_1, x_2$ $c_j - z_j = 0$ - Step - phase 1

phase - original value of obj. var.

RNR

Phase - I - Mini. -> add A.V. Max - subt A.V.

iii) form new obj funn by assigning zero to every original variable

$\therefore$ e.g. $Max = -A_1 - A_2$ $Min = A_1 + A_2$

Phase - II

Consider all original variables

- We have already Removed A.V from Basic table
- change the obj funn for the original coeff
- Then solve the table using simplex method upto optimal solution

$c_j - z_j \geq 0$

e.g

Min $z = x_1 + x_2$ $z_{min} = x_1 + x_2 + 0S_1 + 0S_2$

Sub to $2x_1 + x_2 \geq 4$

$x_1 + 7x_2 \geq 7$

$x_1, x_2 \geq 0$

-> Phase I

z Min $= A_1 + A_2$

$2x_1 + x_2 - S_1 + A_1 = 4$

$x_1 + 7x_2 - S_2 + A_2 = 7$

$S_1, S_2, x_1, x_2 \geq 0$

CBj	cj	0	0	1	1	Solution	Ratio
	B.V	~~X~~$_1$	~~X~~$_2$	~~S~~$_1^n$	A_2		
1	A_1						
1	A_2						

CBj	Cj	0	0	0	0	1	1		
	B V	X_1	X_2	S_1	S_2	A_1	A_2	Sol^n	Ratio
1	A_1	2	1	-1	0	1	0	4	$\frac{4}{1} = 4$
1	A_2	1	$\boxed{7}$	0	-1	0	1	7	$\frac{7}{7} = 1$
	z_j	3	8	-1	-1	1	1		
	$c_j - z_j$	-3	-8 ↑	1	1	0	0		

CBj	Cj	0	0	0	0	1	1		
	B.V	X_1	X_2	S_1	S_2	A_1	A_2	Sol^n	Ratio
1	A_1	$\boxed{13/7}$	0	-1	1/7	1	~~-1/7~~	3	$\frac{3}{\frac{13}{7}} = \frac{21}{13}$
0	X_2	1/7	1	0	-1/7	0	~~1/7~~	1	$\frac{1}{1/7} = 7$
	z_j	13/7	0	-1	1/7	1	~~0~~		
	$c_j - z_j$	-13/7 ↑	0	1	-1/7	0	-		

CBj	cj	0	0	0	0		
	B.V	x_1	x_2	S_1	S_2	Sol^n	Ratio
0	x_1	1	0	$-7/13$	13	$21/13$	
0	x_2	0	1	$1/13$	-2	$10/13$	
	z_j	0	0	0	0		
	$c_j - z_j$	0	0	0	0		

$\therefore$ Phase $c_j - z_j = 0$ $\therefore$ Phase I Complete

Phase - II

CBj	cj	1	1	0	0		
	B.V	x_1	x_2	S_1	S_2	Sol^n.	Ratio
1	x_1	1	0	$-7/13$	13	$21/13$	$21/13/13 = 21$
1	x_2	0	1	$1/13$	-2	$10/13$	$10/13/-2 = \frac{-20}{13}$
	z_j	1	1	$\frac{-6}{13}$	11		
	$c_j - z_j$	0	0	$\frac{6}{13}$	-11		

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	C.F.	1	1	0	0	Sol^n.
C_{Bj}	B.V.	x_1	x_2	s_1	s_2	
1	x_1	$12/7$	$-1/8$	-1	$2/7$	
1	x_2	$7/91$	$7/13$	0	$-7/91$	
	Z_j					
	$C_j - Z_j$					